242. Proposed by J. H. MEYER, S. J., Augusta, Ga.

A given sphere is to be formed into a solid composed of two equal cones on opposite sides of a common base, in such a manner that its surface may be the least possible. Find the dimensions of the solid, and compare its surface with that of the sphere.

Solution by A. H. HOLMES, Brunswick, Maine.

Of the cones into which the given sphere, radius R, is to be transformed, let x=radius of base, and y=altitude.

Then $\dfrac{2\pi x^2y}{3}=\dfrac{4\pi R^3}{3}$ or $x^2y=2R^3$ a minimum, or

$x^4+\dfrac{4R^6}{x^2}$=a minimum.

$\therefore 4x^3=\dfrac{8R^6}{x^3}$, and therefore $x=2^{\frac{1}{6}}R$ and $y=2^{\frac{2}{3}}R$.

Put S_1=surface of sphere, and S_2=surface of required solid.
Then $S_1:S_2=4:2^{\frac{4}{3}}\sqrt{3}$.

Also solved by G. B. M. Zerr.

MECHANICS.

188. Proposed by H. L. ORCHARD, M. A., B. S.

Spherical bubbles of air are rising in water. Find the relation between radius and velocity.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let R=radius of bubble at surface of water, r=radius of bubble at start at bottom, δ=density of gas in bubble referred to water as unity, w= weight of one cubic inch of water in pounds, h=height of column of water equal to weight of one atmosphere, d=depth of water where bubble starts, v=velocity of bubble at distance s from starting point, bubble starting from rest, x=radius of bubble at distance s from starting point, f=acceleration.

$\therefore \frac{4}{3}\pi R^3w\delta$=weight of gas in pounds, $\frac{4}{3}\pi R^3w$=force, in pounds, impelling bubble upwards.

$\therefore f=\dfrac{\frac{4}{3}\pi R^3w(1-\delta)g}{\frac{4}{3}\pi R^3w(1+\delta)}=\dfrac{(1-\delta)g}{1+\delta}$. $\therefore v^2=2fs$. Also $h+d:h+d-s=x^3:r^3$.

$\therefore s=\dfrac{(x^3-r^3)(h+d)}{x^3}$. $\therefore v^2=\dfrac{2f(x^3-r^3)(h+d)}{x^3}$.

$$\therefore \frac{v^2x^3}{x^3-r^3}=2f(h+d)=\frac{2(1-\delta)\ (h+d)g}{1+\delta}$$

d can be found by either method in Vol. I, page 134.

202. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Three equal, uniform, similar rods AB, BC, CD, freely jointed at B and C, are hung from a point by two equal strings attached at A and D. Find the position of equilibrium.

Solution by G. W. GREENWOOD, M. A., Dunbar, Pa.

By symmetry, the strings, of length l, say, make equal angles with the vertical, as do also the rods AB and DC; denote these angles by θ and ϕ, respectively. The rod BC is horizontal. Denote the length of each rod by a, the weight by w, and the depth of the center of gravity of the system below the point of support by z, the strings being regarded as weightless.

$$z=[w(l\cos\theta+\tfrac{1}{2}a\cos\phi)+w(l\cos\theta+a\cos\phi)+w(l\cos\theta+\tfrac{1}{2}a\cos\phi)]/3w$$
$$=\tfrac{1}{3}[3l\cos\theta+2a\cos\phi].$$

For equilibrium, the value of z must be a maximum.

$$\therefore 0=3l\sin\theta\, d\theta+2a\sin\phi\, d\phi\ldots(1).$$

Also, by horizontal projection,

$$a=2l\sin\theta+2a\sin\phi\ldots(2).$$
$$\therefore 0=l\cos\theta\, d\theta+a\cos\phi\, d\phi\ldots(3).$$

$\therefore 3\tan\theta=2\tan\phi$ (by eliminating $d\theta$ and $d\phi$ from (1) and (2)). This equation, with equation (3), gives the position of equilibrium.

Also solved by G. B. M. Zerr and J. Scheffer.

AVERAGE AND PROBABILITY.

187. Proposed by HENRY HEATON, Belfield, N. D.

Through every point of a given square straight lines are drawn in every possible direction, terminating in the sides of the square. What is the average length of such lines?

II. Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let $ABCD$ be the given square, side a. P the randon point coordinates (u, v). On account of the symmetry of the square we will confine P to the triangle ADC. Let EQ be the random line through P, $m=\tan\theta=\tan QED$. For the area AOD, E must fall on HF to intersect opposite sides and on AH to intersect adjacent sides. For the area COD, E must fall on